LAWRENCE
LIVERMORE
NATIONAL
LABORATORY

LLNL-JRNL-557777

Richtmyer-Meshkov instability with viscosity or strength

K. O. Mikaelian

May 17, 2012

Physical Review E

Richtmyer-Meshkov instability with viscosity or strength

Karnig O. Mikaelian

Lawrence Livermore National Laboratory, Livermore, California 94551

We consider a shock passing through the perturbed interface between two viscous or strong materials and present explicit expressions for the amplitude $\eta(t)$. In the linear (nonlinear) regime this 8-parameter problem collapses into 2 (3) non-dimensional variables, leading to scaling laws. We propose a correspondence principle between strength and viscosity, and a new method for measuring viscosity at high pressure and temperature.

PACS numbers 47.20.Bp, 47.20.-k

Viscosity μ and strength Y affect greatly the evolution of surfaces: When a shock hits a piece of metal or a fluid layer with corrugations $\eta(t)\cos(kx)$ on the opposite surface, the perturbations generally change phase then grow with time. Y and μ control the growth. Even if the metal melts ($Y = 0$) it behaves like a viscous fluid ($\mu \neq 0$). Applications include viscous drops [1], high-pressure strength [2], and ejecta [3].

We address questions of scaling: What are the parameters that affect the growth factor $\eta(t)/\eta(0)$? This is an 8-parameter problem: $\eta_0 \equiv \eta(0)$, $k \equiv 2\pi/\lambda$, densities ρ^h and ρ^l (h=heavy, l=light), viscosities μ^h and μ^l (or strength Y^h and Y^l), jump velocity $\Delta\mathrm{v}$, and time t. We present an explicit expression for $\eta(t)$ finding that the problem collapses into two or three variables only: an appropriately defined Reynolds number and a non-dimensional time. The third variable, needed only in the nonlinear regime, is $\eta_0 k$. We answer questions such as: Does the growth depend on the product, sum, or some other combination of μ^h and μ^l? *Ditto* for Y. Can we use this viscosity- or strength-mitigated Richtmyer-Meshkov (RM) instability [4] to measure μ or Y?

Viscosity. The first linear treatment of the viscous RM instability was based on approximate eigenvalues and gave [5]

$$\eta(t) = \eta_0 + \dot{\eta}_0(1 - e^{-2\nu k^2 t})/2\nu k^2 \tag{1}$$

where $\dot{\eta}_0 = \eta_0 \Delta \mathrm{v} k A$ is the inviscid growth rate, $A \equiv (\rho^h - \rho^l)/(\rho^h + \rho^l)$, and $\nu \equiv (\mu^h + \mu^l)/(\rho^h + \rho^l)$. Eq. (1) asymptotes to

$$\eta_\infty = \eta_0 + \dot{\eta}_0 / 2\nu k^2 = \eta_0(1 + \Delta \mathrm{v} A / 2\nu k). \tag{2}$$

Subsequently, an alternative expression was presented [6]:

$$\eta(t) = \eta_0 + \dot{\eta}_0 t(1 - C t^{1/2}) \tag{3}$$

where $C \equiv 16k\sqrt{\mu^h \mu^l \rho^h \rho^l}/[3\sqrt{\pi}(\sqrt{\mu^h \rho^h} + \sqrt{\mu^l \rho^l})(\rho^h + \rho^l)]$. No derivation was given but it is clear that Eq. (3) applies only to limited cases where $C \neq 0$: It cannot be used when $\rho^h = 0$ or $\rho^l = 0$, or when $\mu^h = 0$ or $\mu^l = 0$ because C depends on their *product*. In contrast, ν in Eq. (1) depends on the *sum* and therefore viscous effects persist as long as $\mu^h + \mu^l \neq 0$.

Eq. (3) has another limitation: It can apply for "early" times only. It may be used, if at all, only for $t < 4/9C^2$ because it gives $d\eta/dt = 0$ at $t = 4/9C^2$ and becomes negative (!) for $t > 4/9C^2$, clearly unphysical.

Despite these shortcomings we found a few cases where Eq. (3) did better than Eq. (1) at early times. Our procedure was to compare Eq. (1), Eq. (3), and numerical simulations with the hydrocode CALE [7] solving the full Navier-Stokes equations. At present CALE can treat only constant viscosities but this was enough for our purpose. These simulations confirmed the above statements: Viscous effects persist even when only one of the fluids has viscosity, the controlling parameter is ν, and $\dot{\eta}$ does not reverse sign at any time. But there were also cases, mostly with $\mu^h \sim \mu^l$, where Eq. (3) did better than Eq. (1) at early times.

Our first attempt to improve upon Eq. (1) was to use exact eigenvalues. In general, when one of them vanishes (say $\gamma_+ = 0$) the result is

$$\eta(t) = \eta_0 - \dot{\eta}_0(1 - e^{\gamma_- t})/\gamma_- . \tag{4}$$

In the approximation of [5] $\gamma_- = -2\nu k^2$, hence Eq. (1). Using exact eigenvalues we find (details elsewhere) that $\gamma_+ = 0$ still, but γ_- is different. To our surprise, however, using exact eigenvalues gave substantially *worse* results than Eq. (1). For example, for the simplest, one-fluid case we find $\gamma_- = -2[4 + (\sqrt{297} - 17)^{1/3} - (\sqrt{297} + 17)^{1/3}]\nu k^2/3 \approx -0.9126\nu k^2$. In fact the equation $\eta(t) = \eta_0 + \dot{\eta}_0(1 - e^{-0.9126\nu k^2 t})/0.9126\nu k^2$ can be found in the summary by Bakhrakh *et al.* [8]. Its asymptotic growth is more than twice larger and completely ruled out by our numerical simulations.

The only remaining option to improve upon Eq. (1) is to treat the problem as an initial-value problem, similar to the viscous RT instability [9]. The initial-value approach is substantially more complex and to date there are no exact results valid for arbitrary

$\mu^{h,l}$. We have succeeded, however, in deriving an exact and general expression for the asymptote η_∞ and *the result is Eq. (2)*. As for $\eta(t)$, the general Laplacian which must be inverted is too complicated to carry out analytically. We found, however, the following expression to be extremely accurate in describing our CALE results:

$$\eta(t) = \eta_0 + \frac{\dot{\eta}_0}{4\nu k^2} erfc(-\sqrt{\tau}) + \frac{\dot{\eta}_0}{\nu k^2} \sum_{i=2}^{4} \frac{Z_i}{D'(Z_i)} e^{(Z_i^2-1)\tau} erfc(-Z_i\sqrt{\tau}) \tag{5}$$

where $erfc(z)$ is the complimentary error function $1 - err(z)$, $\tau \equiv \nu k^2 t$, $Z_2 = 1/4$, $Z_{3,4} = (-7 \pm 4i\sqrt{11})/9$, $D'(Z_2) = -155/64$, and $D'(Z_{3,4}) = 8(1111 \mp 28i\sqrt{11})/729$. Together with $Z_1 = 1$, $D'(1) = 4$, they satisfy the four sum rules $\sum_{i=1}^{4} 1/D'(Z_i) = \sum_{i=1}^{4} Z_i/D'(Z_i) = \sum_{i=1}^{4} Z_i^2/D'(Z_i) = 0$, and $\sum_{i=1}^{4} Z_i^3/D'(Z_i) = 1$.

Eq. (5) has several surprising properties: the Atwood number A does not appear in it except for $\dot{\eta}_0 = \eta_0 \Delta \mathrm{v} k A$. It is an exact expression only if $\nu^h = \nu^l$ ($\nu^{h,l} \equiv \mu^{h,l}/\rho^{h,l}$), but it is also an extremely good approximation for arbitrary $\nu^{h,l}$. Its asymptote, η_∞, agrees with Eq. (2), the only exact formula for arbitrary $\nu^{h,l}$. What is surprising is that there are actually infinitely many solutions, each associated with a different A and all giving approximately the *same* result, within a few percent, which is the reason why A does not appear in Eq. (5). In the exact solution for $\nu^h = \nu^l$ the 6 constants Z_i and $D'(Z_i)$, $i = 2,3,4$ are determined from

$$Z^3 + (2 - A^2)Z^2 + (1 + 2A^2)Z - A^2 = 0, \tag{6a}$$

$$D'(Z) = 4Z^3 + 3(1 - A^2)Z^2 + 2(3A^2 - 1)Z - 1 - 3A^2. \tag{6b}$$

The solution we chose corresponds to $A = 5/6$ because it gives particularly simple expressions for Z_i and $D'(Z_i)$ quoted above.

We illustrate with an example: $\rho^h = 4\,\text{g/cm}^3$, $\rho^l = 1\ \text{g/cm}^3$, $\Delta\text{v} = 0.1\,\text{cm/ms}$, $\lambda = 2.5$ cm, $\eta_0 = 30\,\mu\text{m}$, $\mu^h = 0.32\,\text{Pa-s}$, and $\mu^l = 2\mu^h$. Fig. 1 shows $\eta(t)$ as calculated by Eqs. (1), (3), (5), and CALE. Eq. (1) overestimates $\eta(t)$ at early times but its asymptote, Eq. (2), is reproduced by Eq. (5) and by CALE. Eq. (3) shows better agreement with CALE but only at early times – its behavior after 89 ms ($4/9C^2$) is unphysical. Only Eq. (5) agrees with CALE both at early and late times. The reader should be surprised at this because Eq. (5) is exact only for $\nu^l = \nu^h$ and $A = 5/6$ while in this example $\nu^l = 8\nu^h$ and $A = 3/5$. We repeat that other solutions to Eqs. (6a,b) give essentially the same $\eta(t)$ when substituted in Eq. (5). We will present elsewhere another exact expression for the case $\mu^h = 0$ or $\mu^l = 0$ and arbitrary A. Again, we find it well duplicated by Eq. (5).

Eq. (5) displays simple scaling: Out of the eight independent variables $\eta_0, k, \rho^h, \rho^l, \Delta\text{v}, \mu^h, \mu^l, t$, only two combinations are relevant: $\Delta\text{vA}/\nu\text{k}$ and $\nu\text{k}^2 t$. It is customary to define a Reynolds number as a ratio of inertial to viscous forces. We propose $\text{Re} = |\Delta\text{v}|\text{A}/\nu\text{k}$ so that $\eta(t)/\eta_0 = f(\pm\text{Re}, \tau)$. Eq. (2) reads $\eta_\infty = \eta_0(1 \pm \text{Re}/2)$. Note that $\eta_\infty = 3\eta_0$ or $-\eta_0$ when $\text{Re} = 4$, a case discussed below.

Strength. We now consider shocks in ideal elastic-plastic solids characterized by a constant shear modulus G and yield strength Y. Early work, primarily experimental, is summarized in [8]. Here we follow-up on the suggestion [10] that strength may be treated as viscosity because they found that using a $\mu - Y$ relationship in our analytic viscous formulae gave reasonable results for RT strength experiments [10]. Other strength experiments have also been analyzed in terms of viscosity [2]. We presented simulations for possible RM experiments proposing them as a measure of strength [11], and recent experiments [3] confirm the viability of this approach: following shock break-out $\eta(t)$ depends on Y and grows larger (smaller) in weaker (stronger) metals. There are strength effects which cannot be duplicated by viscosity [12]: Cut-off amplitudes and wavelengths below which perturbations do not grow, known as the Drucker and Miles limits. Both are proportional to $1/g$ and vanish in the RM case where $g \to \infty, \int g dt = \Delta \mathrm{v}$, yielding $\dot{\eta}_0 = \eta_0 \Delta \mathrm{v} k A$ as in the fluid case. See also [13].

We find that no $\mu - Y$ relationship can provide exact agreement between η^Y and η^μ – only a qualitative agreement can be obtained, within 30-40%, made possible by two opposing trends: η^Y grows faster but saturates earlier, while η^μ grows slower but saturates later. The following relationship

$$Y \approx 2|\dot{\eta}_0| k\mu / 3 \tag{7}$$

provides that qualitative agreement. Eq. (7) means that η between two fluids of viscosities $\mu^{h,l}$ will evolve similar to the case of two metals whose yield strengths $Y^{h,l}$ satisfy Eq. (7). Actually, only the sum $Y^h + Y^l$ is important. We have verified this by direct numerical simulations.

A comparison between η^μ (in black) and η^Y (in red) is given in Fig. 2. The lower curves refer to the same problem as in Fig. 1, so the same CALE curve is reproduced in black. In red is the problem with strength where $Y^{h,l}$ are related to $\mu^{h,l}$ by Eq. (7): $Y^h = 0.24 Pa$, $Y^l = 2Y^h = 0.48 Pa$. The shear moduli are taken to be 10^3 times larger – they control mostly the oscillations after η^Y reaches its maximum [8, 13]. The two upper curves in Fig. 2 refer to the same problem but with the shock generated in the heavy fluid inducing the same $|\Delta \mathrm{v}|$, now taking $\mu^l = 0$. There is growth after the phase reversal ($\Delta \mathrm{v}$ and hence $\dot{\eta}_0$ are negative), and a reshock occurs at 270 ms, just as η^{μ} and η^{Y} cross. The inset shows the two interfaces at this time; they have the same amplitude but the shape of the Y–problem is triangular. This difference in shape persists after reshock but the amplitudes continue to track each other.

Needless to say, Eq. (7) does *not* mean that strength depends on η_0, $\Delta \mathrm{v}$, k, etc.! It is only a correspondence principle to convert μ–derived results to Y. Substituting it in Eq. (2) one obtains

$$\eta_\infty = \eta_0 + \dot{\eta}_0 |\dot{\eta}_0| (\rho^h + \rho^l) / 3k(Y^h + Y^l) \tag{8}$$

to be compared with $\eta_0 + 0.29 \rho \dot{\eta}_0^2 / kY$ for a single fluid [13]. Let us apply Eq. (8) to the case $\eta_\infty = -\eta_0$, i.e. where the perturbation stops growing after a complete phase change. For strength, the requirement in a single fluid is

$$Y_{\eta_\infty = -\eta_0} = \eta_0 (\Delta \mathrm{v})^2 k\rho / 6. \tag{9}$$

As shown in Fig. 12 of [11], this happens for the SG model in Aluminum. Using $\rho = 2.7$ g/cm^3, $\eta_0 = 0.02$ cm, $\lambda = 1$ cm, $\Delta \mathrm{v} = 3.5/15 = 0.23$ cm/μs (all taken from [11]), the rhs of Eq. (9) gives 3 kb, agreeing with the Y_0 (2.9 kb) of SG. This after-the-fact comparison builds confidence that Eq. (7) is a reasonable relationship. A similar relation appears to work for the RT case also where the inverse time scale $|\dot{\eta}_0| k$ is replaced by $\sqrt{gkA}$, as long as the amplitude and wavelengths are above the cut-offs.

Translating the viscous Reynolds number into strength reads $\mathrm{Re} = 2\dot{\eta}_0 \Delta \mathrm{v} A(\rho^h + \rho^l)/3(Y^h + Y^l)$. For reference, Fig. 1 as well as the two lower curves in Fig. 2 have $\mathrm{Re} \approx 13$, while the two upper curves in Fig. 2 have $\mathrm{Re} \approx 38$.

Compressibility. The theory and simulations discussed so far have been limited to incompressible fluids – we used an ideal equation-of-state with a high adiabatic index so the densities change very little. An exact treatment of the compressible RM problem with viscosity or strength has never been carried out and is likely to be a daunting task – repeat Richtmyer's calculations adding viscosity or strength. Instead, we look for "prescriptions" which approximately account for compressibility. This was done first by Richtmyer for the case when a shock is reflected: Use η_{after} for η_0 and A_{after} for A in the incompressible result $\eta(t) = \eta_0(1 + \Delta \mathrm{v} k A t)$. When a rarefaction is reflected Meyer and Blewett [14] prescribed $(\eta_{before} + \eta_{after})/2$ and A_{after}. Naturally we keep these prescriptions because we should recover them in the inviscid limit. Hence the only question is: What viscosity should be used in Eqs. (1-5)?

We ran problems with appreciable compressibility and compared the results with Eq. (5). Best agreement was obtained when ν_{after} was used, and this is what we recommend and is probably expected since η evolves after the passage of the shock.

In our simulations we could use only constant $\mu^{h,l}$ but of course $\rho^{h,l}$ increased and therefore ν changed (by ~60% in our compressible examples). This implies that, all other things being equal, compressibility *increases* the viscous growth rate because $\nu \sim \mu/\rho$ decreases. The same effect will arise when shocks heat the fluids and, in general, reduce their viscosities.

The only method proposed so far to measure viscosities at high pressures and temperatures is the "Sakharov method" reviewed extensively in [15]: Measure the decay of a corrugated shock in a viscous fluid. We believe the viscous RM instability is a more effective way because $\eta_\infty \propto 1/\nu$ from Eq. (2). The growth depends on the sum of the viscosities on either side of the interface, but choosing one of the fluids to be inviscid isolates the viscosity of the other. An example was shown in Fig.2: $\mu^l = 0$ hence $\nu = \mu^h/(\rho^h + \rho^l)$. We have verified, by numerical simulations, that the method proposed for strength [11] works equally well with viscosity.

Nonlinearity. Layzer's nonlinear model for a single inviscid fluid [16] and its extension to two fluids [17] are natural candidates for a nonlinear viscous model – keep the viscous term in the Bernoulli equation. This was done by Sohn [18]. However, we find that this model is even more limited than the inviscid model, the limitations and failures of which were reported in [19].

(We should point out that in the linear limit this viscous model reduces to our model and that Sohn's linear RM solution (Eq. (11) in [18]) is in error - the correct $\eta(t)$ was given in [5], reproduced here as Eq. (1)).

We find that the model gives reasonable results only for the bubble and only for $A = 1$. If $A \neq 1$ the model predicts "negative viscosity" for large initial amplitudes. Thus we concentrate on the original single-fluid Layzer model augmented by viscosity:

$$(2\eta_2 + ck/2)(\ddot{\eta} + 2\nu k^2\dot{\eta}) + c^2k^2\dot{\eta}^2/4 + 2g\eta_2 = 0, \tag{10}$$

where $\nu = \mu/\rho$ is the kinematic viscosity of the fluid, $\eta_2(t) = -ck\{1 + [(1+c)\eta_0 k - 1]e^{-k(1+c)(\eta-\eta_0)}\}/4(1+c)$, and $c = 1(2)$ for 3D(2D), as in the inviscid case [19].

Eq. (10) can be solved analytically by the (η^*, t^*) technique: Use the linear solution until $\eta = \eta^* \equiv 1/(1+c)k$ followed by the nonlinear solution (given below). Setting $g = 0$ (this is not necessary – we will consider the RT problem elsewhere) we find

$$\eta(t) = \eta_0 + [2/(k+ck)]\ln[1 + (1+c)(\dot{\eta}_0 / 4\nu k)(1 - e^{-2\nu k^2 t})] \tag{11}$$

confirming again that "the nonlinear solution is essentially the logarithm of the linear solution" – compare with Eq. (1).

From Eq. (11) the nonlinear asymptote is

$$\eta_\infty = \eta_0 + [2/(k+ck)]\ln[1 + (1+c)\dot{\eta}_0 / 4\nu k] \tag{12}$$

to be compared with Eq. (2). Combining this with Eq. (7) we obtain

$$\eta_\infty = \eta_0 + [2/(k+ck)]\ln[1 + (1+c)\dot{\eta}_0^2 \rho / 6Y]. \tag{13}$$

Setting $c = 2$ this equation agrees quite well with the asymptotic 2D bubble amplitudes computed by Dimonte *et al*. (Fig. 2 in Ref. [3]).

How about the spike? Zhang [20] proposed using the Layzer model with $\eta < 0$ for spikes, and indeed this works for the inviscid spike when $A = 1$ [19]. However, we find that the viscous model is a poorer representation of the spike when we solve Eq. (10) numerically with $\eta_0 < 0$.

Fig. 3 illustrates the above observations. The problem is the same as the He/Xe problem used previously for its large Atwood number, $A \approx 0.94$, adding viscosity (black curves) or strength (red curves) to the heavy "Xe", using Eq. (7) for the $\mu - Y$ correspondence. These 4 curves, calculated by 2D CALE, are compared with the numerical solution of Eq. (10) for the spike and with the analytic solution, Eq. (11), for the bubble. The initial amplitude is 0.7 cm and $\lambda = 13$ cm, so $\eta_0 \approx \eta^* = 1/3k$, and

$\Delta v = 8.25$ cm/ms. For viscosity we chose $\mu = 0.2$ Pa-s giving $\mathrm{Re} = 45$. From Eq. (7) the corresponding Y is ≈ 160 Pa. The inset shows the interfaces as calculated by CALE at $t = 6$ ms when they have moved 50 cm. The $\mu - Y$ correspondence does better for the bubble than for the spike in the nonlinear regime (there is no bubble/spike difference in the linear regime). Eq. (11) is a good model for the bubble, but the spike is overestimated. This may be due to 1) $A = 1$ in Eq. (10), 2) we have used v_{after}, and 3) nonlinear effects suppress $\dot{\eta}_0$. Note that the asymptotic spike satisfies $\eta_2 \to \infty$ and, from Eq. (10), $\ddot{\eta} + 2\nu k^2 \dot{\eta} = 0$, a particularly simple equation expressing conservation of $\dot{\eta} + 2\nu k^2 \eta$.

Conclusions. The viscous RM instability in the linear and incompressible regime is well described by Eq. (5) and, to a lesser degree, by the much simpler Eq. (1). Eq. (2) is exact. The viscous compressible problem remains challenging and untackled even in the linear regime. The prescription to use postshock viscosities appears adequate.

Nonlinear *and* compressible viscous problems must clearly be solved numerically. We were hoping that the nonlinear *incompressible* problem could be amenable to modeling [18], but only the $A = 1$ (single fluid) bubble appears reasonable – See Fig. 3 and Eq. (11).

The $\mu - Y$ surrogacy is approximate and based only on similarity of $\eta^{\mu}(t)$ and $\eta^{Y}(t)$ if μ and Y satisfy Eq. (7). Note that μ and Y have opposite dependence on wavenumber: $\eta_{\infty}^{\mu} \sim 1/k$ while $\eta_{\infty}^{Y} \sim k$, so that doubling the wavelength will essentially double η_{∞}^{μ} but cut η_{∞}^{Y} in half, confirmed by simulations also.

Generally, we find μ to have a weaker effect in nonlinear problems, which can be understood by comparing Eqs. (2) and (12): $\eta_{\infty} \sim 1/\nu$ in the linear regime, but $\sim \ln(1/\nu)$

in the nonlinear regime. Similarly for Y. RM experiments with viscosity, as an alternative to the "Sakharov method", will be more discriminating with small η_0. We hope our findings will spur further experimentation.

This work was performed under the auspices of the U. S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

REFERENCES

[1] T. G. Theofanous, Ann. Rev. Fl. Mech. **43**, 661 (2011).

[2] H.-S. Park *et al.*, Phys. Rev. Lett. **104**, 135504 (2010).

[3] G. Dimonte *et al.*, Phys. Rev. Lett. **107**, 264502 (2011).

[4] R. D. Richtmyer, Commun. Pure Appl. Math. **13**, 297 (1960); E. E. Meshkov, Fluid Dyn. **4**, 101 (1969).

[5] K. O. Mikaelian, Phys. Rev. E **47**, 375 (1993).

[6] P. Carlès and S. Popinet, European J. Mech. B Fluids **21**, 511 (2002).

[7] R. E. Tipton, CALE Users Manual (unpublished).

[8] S. M. Bakhrakh *et al.*, UCRL-CR-126710 (1997) (unpublished).

[9] A. Prosperetti, Phys. Fl. **24**, 1217 (1981).

[10] J. D. Colvin *et al.*, J. Appl. Phys. **93**, 5287 (2003).

[11] K. O. Mikaelian, Phys. Fl. **17**, 034101 (2005).

[12] A. R. Piriz *et al*., Phys. Rev. Lett. **105**, 179601 (2010); H.-S. Park *et al*., *ibid* **105**, 179602 (2010).

[13] A. R. Piriz *et al*., Phys. Rev. E **78**, 056401 (2008).

[14] K. A. Meyer and P. J. Blewett, Phys. Fl. **15**, 753 (1972).

[15] G. H. Miller and T. J. Ahrens, Rev. Mod. Phys. **63**, 919 (1991).

[16] D. Layzer, Astrophys. J. **122**, 1 (1955).

[17] V. N. Goncharov, Phys. Rev. Lett. **88**, 134502 (2002).

[18] S.-I. Sohn, Phys. Rev. E **80**, 055302 (2009).

[19] K. O. Mikaelian, Phys. Rev. E **78**, 015303 (2008).

[20] Q. Zhang, Phys. Rev. Lett. **81**, 3391 (1998).

Figure Captions

Fig. 1. Comparison of Eqs. (1), (3), and (5) with a CALE simulation of the problem discussed in the text. The asymptotic value η_∞ is 218 μm from Eq. (2).

Fig. 2. Four growth factors calculated by CALE: black for μ and red for Y. The lower curves refer to the same problem as in Fig. 1. The upper curves refer to a problem with Δv replaced by $-\Delta \text{v}$ (shock generated in the heavy fluid), with $\mu^l = 0$. Reshock occurs at 270 ms. The inset shows the interfaces for the μ and Y problems

at 270 ms, the vertical scale greatly enhanced for clarity. In both problems $Y^{h,l}$ and $\mu^{h,l}$ are related by Eq. (7).

Fig. 3. CALE calculation of bubbles (lower curves) and spikes (upper curves) for the $A \approx 1$ problem discussed in the text, black referring to μ and red to Y, related by Eq. (7). Blue dashed lines are from Eq. (10), solved numerically with $\eta_0 = -0.7\,\mathrm{cm}$ for the spike, and from Eq. (11) for the bubble. The inset shows the interfaces for the μ and Y problems at $6\,\mathrm{ms}$. The asymptotic value of the bubble is 4.1 cm from Eq. (12) or (13).

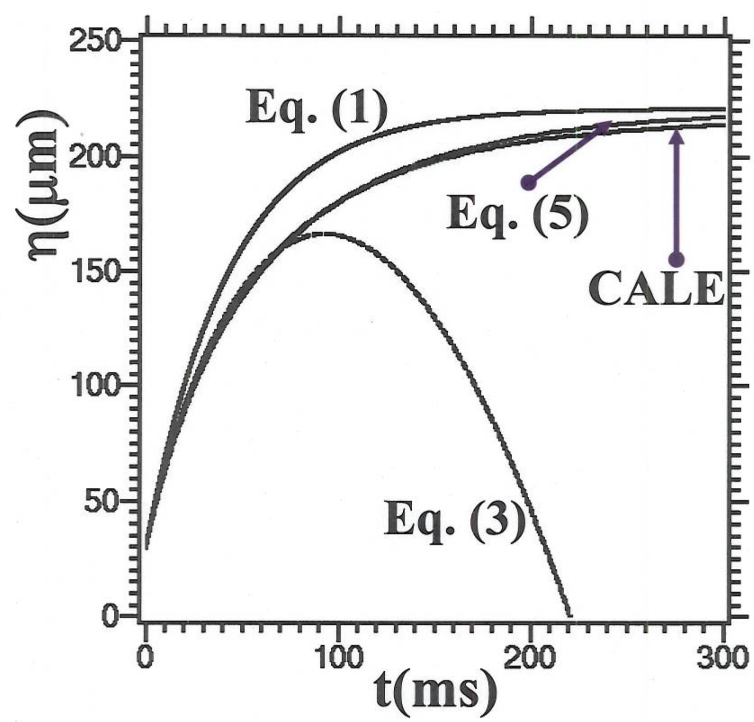

Fig. 1

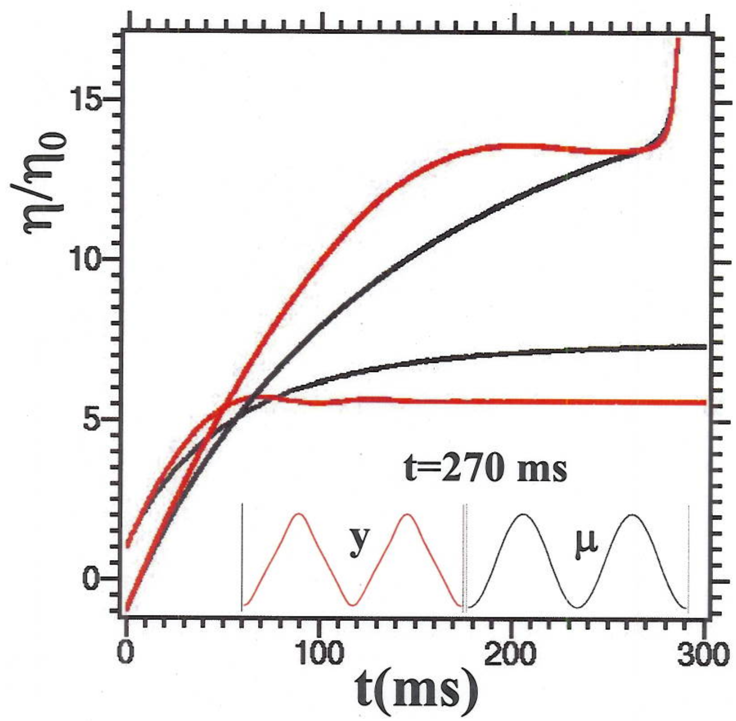

Fig. 2

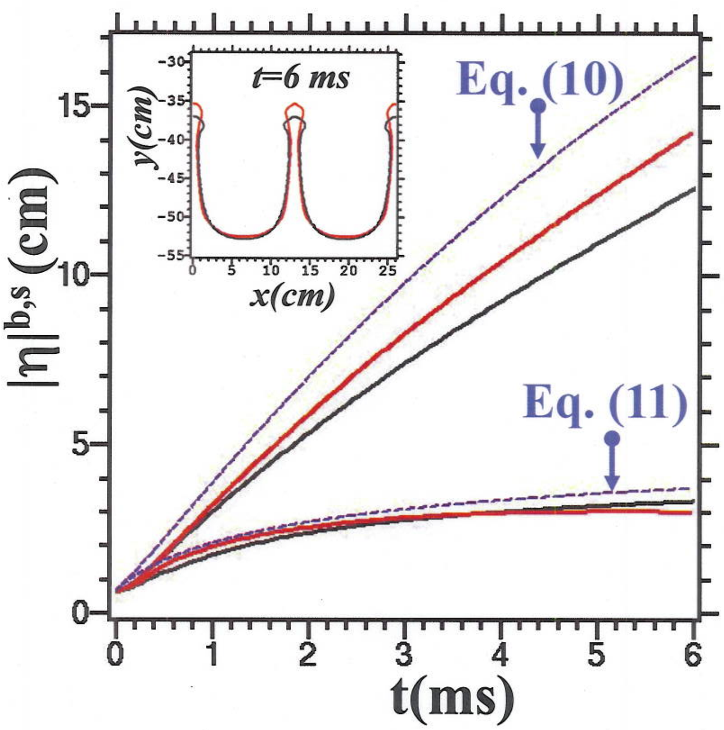

Fig. 3